ference in Π of the two samples in the region where only the first phonon contributes. With $\beta_1/\beta_3=(m^*_1/m^*_3)^{1/2}$ we obtained $\beta_1=16$ and $\beta_3=18$, both values quoted for $V=0$.

Figure 7 shows the comparison of Eq. (17) with the difference in Π of the two samples. Although the structure of the bias dependence is reproduced quite well by the theoretical curve, the agreement is only qualitative. The effect of the shift in valley population can clearly be seen in the region between 0.008 and 0.030 V bias. The discrepancy becomes large, however, as soon as the higher energy phonon starts to contribute to the tunneling current. Here the experimental data fall off abruptly from the theoretical curve. There is no known reason why the two phonons should behave differently, but in order to see what would happen, a curve was plotted as though the valleys were equivalent for tunneling involving the higher energy phonons. This curve is shown dashed in Fig. 7. As can be seen from the forward bias data, this modification is too drastic. The data in the reverse bias direction are in better agreement, but it is hard to imagine any mechanism which would distinguish forward from reverse bias. The falloff of the data in the reverse bias direction will be affected by exactly the same smearing out mechanisms which are active beyond the Kane kink. If these mechanisms are indeed responsible for the slow falloff in that region, similar behavior in this bias region is to be expected also. This smearing would not, however, account for the fact that the observed difference between the Π of the two samples near $V=-0.13$ V is appreciably smaller than the calculated shear coefficient. We, therefore, conclude that the variation of the inherent tunneling probability per electron is not as sensitive to shear as would be expected. Within the framework of the theory the only possibilities which can account for this are: (a) that the chosen value for β is too low; (b) that the effect of shear on the reduced effective mass is quite large and in the opposite direction as the effect of the valley energy shifts.

There is no value of β which is simultaneously consistent with the shear data and the hydrostatic pressure data. In view of the general agreement between the various independent calculations of β, possibility (a) is unlikely. Possibility (b) can be eliminated also because of the similarity of Π for sample 1 and Π_p. Furthermore, the mechanism responsible for the discrepancy beyond 30 mV in the forward bias region is undoubtedly also operating in this bias region. It, therefore, seems that a different mechanism not accounted for by the present theory contributes to the shear stress coefficient.

C. Comparison of Absolute Magnitudes of Tunneling Parameters with Theory

The physical properties of a diode determine the stress coefficient through the parameters α and β. These parameters could not be determined unambiguously

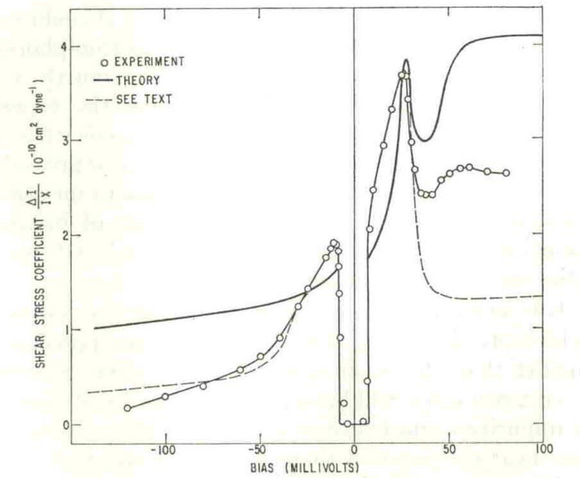

FIG. 7. The difference between the uniaxial stress coefficients of samples 1 and 2. Comparison between theory and experiment. The heavy curve represents the theory based on the anisotropy of the tunneling probability in the whole bias range of indirect tunneling. The dotted curve was calculated assuming that the (111) valleys are equivalent for tunneling involving the higher energy phonon.

from the stress coefficients because the stress-induced changes of the relevant effective masses are not known. These measurements would yield these effective mass changes if it were possible to determine the parameters α and β by some other method. The theory leading to Eqs. (1) and (2) does not determine these parameters accurately enough since it is based on the assumption of a constant junction field and neglects the spatial distribution and fluctuations of the impurity concentrations near the junction.

It is, nevertheless, interesting to compare and evaluate the values of α and β predicted by the constant junction field theory with the values obtained above from the stress coefficients using reasonable estimates for $\Delta m^*/m^*$. The coefficients α and β can be obtained from the theory, (i) by direct computation using Eqs. (1), (2), and (3), and (ii) by calculating C and D and using the measured I for a given bias voltage. This was done with the following choice of parameters appropriate for our diodes: $\zeta_p=0.15$ eV, $\zeta_n=0.020$ eV, $p=6\times10^{19}$, $n=5.5\times10^{18}$, junction area $=0.002$ cm^2, $\kappa=16$, $I_d=0.370$ A at $V=-300$ mV.

The values for α and β obtained by method (i) are listed in the first row of Table I. The third row gives α

TABLE I. Values of tunneling exponents α and β.

	α	Bias (mV)	β	Bias (mV)
Theory	8.5	−300	12.5	−300
Theory (see text)	12	−300	17.5	−300
Theory and I_d	12.5±1	−300		
Pressure exp.	17.6	−300	16±3	+60
Pressure exp.			20±4	−70
Shear exp.			17±1	0

as calculated by method (ii). The value of β could not be obtained in this way because the electron-phonon coupling constants are not known. The fourth and fifth rows list the values obtained from the experimentally observed hydrostatic pressure coefficients as explained in the previous section. The last row gives the value of β used to fit the shear contribution to the stress coefficient of sample 2 in the bias range of indirect tunneling. The bias voltages at which the listed quantities were obtained are included in Table I.

It is seen that α and β as computed from the constant field expressions Eqs. (1), (2), and (3) are appreciably smaller than the other values. This is to be expected since a real diode will have a more diffused distribution of impurities, and hence, a smaller impurity concentration near the junction than in the bulk material. This yields a wider junction. Following Meyerhofer *et al.*,[23] we assume the donor concentration near the junction to be half of that of the bulk and obtain by direct computation of α and β the values quoted in the second row of Table I.[24]

The uncertainty of the experimental values stems from the difficulty of estimating $\Delta m^*/m^*$ and also from the fact that for the pressure coefficients of the energy gaps the values measured on pure material at 300°K were used. It is hoped to obtain the temperature dependence of these pressure coefficients by studying the stress tunneling coefficients at elevated temperatures.

V. SUMMARY AND CONCLUSION

The magnitude and the orientation dependence of the effect of stress on the tunneling current enables one to distinguish three different tunneling processes in Sb-doped germanium.

(1) In the small bias range $-8\text{ mV}<V<+8\text{ mV}$ the tunneling is direct. The detailed nature of the tunneling process in this range is not known. The current is not affected by the relative shifts of the (111) valleys under shear and hence cannot arise from a sum of independent (111) valley contributions.

(2) Beyond about 8 mV at helium temperatures, but before the onset of direct tunneling into the (000) conduction band, the tunneling process is phonon assisted. Here a large additional contribution to the stress coefficient is observed for a shear stress which lifts the degeneracy of the (111) valleys when the orientation of the junction field is such that the effective mass components of the valleys in the field direction are different. This demonstrates clearly the anisotropy of the tunneling probability when the effective mass is anisotropic as predicted by the theory.

(3) For biases $V<-140$ mV, direct tunneling into the (000) conduction band is observed. Near the onset voltage V_k the magnitude of the stress coefficients increases sharply because of the pressure dependence of V_k. This sharp increase allows an accurate determination of $E_g(000)-E_g(111)$, the energy separation between the conduction band edges at (000) and (111), respectively. We find for this the value 0.160 ± 0.005 eV which is about 0.015 eV larger than that for pure material. This larger value of the conduction band separation agrees with recent results of infrared absorption measurements[16] in degenerate germanium. The sharp rise of the stress coefficients near $V=V_k$ indicates that the (000) conduction band edge is quite well defined despite the large impurity concentration.

The major features of the bias and orientation dependencies of the stress coefficients agree with the present theory of tunneling. There are, however, several interesting observations which remain unexplained at present. One is the structure in the bias dependence of the pressure coefficient Π_p. The apparent value of the tunneling exponent β is approximately 25% larger in the *LA* phonon region than in the *TA* phonon region. The second is the disagreement between the theoretically predicted shear contribution to Π with the experiments in the bias range where *LA* phonons can be emitted. Apparently the way in which the phonons are incorporated into the theory does not adequately describe their differences. The third observation which needs some further study is the bias dependence of the indirect tunneling exponent β which is found to be opposite to the prediction of theory.

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[23] D. Meyerhofer, G. A. Brown, and H. S. Sommers, Phys. Rev. **126**, 1329 (1962).

[24] Nathan discussed and employed a method for obtaining β from the bias dependence of the reverse current at room temperature (see reference 20). Although several aspects of the shape of the *I-V* characteristic still remain unexplained we have used his method and find for our samples at zero bias $\beta=18\pm1$ using his best-fit parameter $\gamma=1.25$.